AP Calculus BC

Course Description:

The curriculum for this elective Advanced Placement course is developed from the College Board AP Curriculum for BC Calculus. AP Calculus BC is roughly equivalent to both first and second semester college calculus courses. Students will be exposed to and practice skills related to: extension of the content learned in AB to different types of equations (polar, parametric, vector-valued) and new topics (Euler's method, integration by parts, partial fraction decomposition, and improper integrals); introduces the topic of sequences and series; topics in differential and integral calculus (limits, derivatives, definite integrals, the Fundamental Theorem of Calculus, and series); a variety of representations (graphically, numerically, analytically, and verbally) and to make connections amongst these representations; how to use technology (solve problems, experiment, interpret results, and support conclusions). Grades are determined by formative questions and summative exams. Successful completion of the AP Exam in Calculus BC, a score of 3 will result in college credit earned for Calculus AB and a score of 4 or 5 will result in college credit earned for Calculus BC. This information in this course overview outlines what students should understand and be able to do by the end of two trimesters.

Mastery Standards:

As written from College Board AP Curriculum for Calculus BC

Limits

Students must have a solid, intuitive understanding of limits and be able to compute one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to apply limits to understand the behavior of a function near a point and understand how limits are used to determine continuity.

Standard 1: Students will understand that the concept of a limit can be used to understand the behavior of functions.

Standard 2: Students will understand that continuity is a key property of functions that is defined using limits.

Derivatives

Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. Students should also be able to solve separable differential equations, understand and be able to apply the Mean Value Theorem, and be familiar with a variety of real-world applications, including related rates, optimization, and growth and decay models.

Standard 3: Students will understand that the derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.

Standard 4: Students will understand that a function's derivative, which is itself a function, can be used to understand the behavior of the function.

Standard 5: Students will understand that the derivative has multiple interpretations and applications including those that involve instantaneous rates of change.

Standard 6: Students will understand that the Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval.

Integrals and the Fundamental Theorem of Calculus

Students should be familiar with basic techniques of integration, including basic antiderivatives and substitution, and properties of integrals. Students should also understand area, volume, and motion applications of integrals, as well as the use of the definite integral as an accumulation function. It is critical that students understand the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus.

Standard 7: Students will understand that antidifferentiation is the inverse process of differentiation.

Standard 8: Students will understand that the definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.

Standard 9: Students will understand that the Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.

Standard 10: Students will understand that the definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.

Standard 11: Students will understand that antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change.

Series

Students should be familiar with various methods for determining convergence and divergence of a series, Maclaurin series for common functions, general Taylor series representations, radius and interval of convergence, and operations on power series. The technique of using power series to approximate an arbitrary function near a specific value allows for an important connection back to the tangent-line problem.

Standard 12: Students will understand that the sum of an infinite number of real numbers may converge.

Standard 13: Students will understand that a function can be represented by an associated power series over the interval of convergence for the power series.

Unit	Description of Unit and Learning Targets
Unit 1: Limits and Continuity Essential Questions: ● Can change occur at an instant? ● How does knowing the value of a limit, or that one does not exist, help you make sense of interesting features of functions and graphs?	Students will……. Learning Targets: ● Understand limit notation and can evaluate limits graphically, numerically, and algebraically. ● Understand definition of continuity and can remove discontinuities. ● Interpret limits involving infinity, asymptotic behavior, and using the squeeze theorem. ● Draw conclusions about a functions behavior on an interval using Intermediate Value Theorem and Extreme Value Theorem.
Unit 2: Differentiation: Definition and Basic Derivative Rules Essential Questions: ● How is the average rate of change related to instantaneous rate of change? ● How are derivatives related to continuity and the tangent line on a curve?	Students will... Learning Targets: ● Find average and instantaneous rates of change and evaluate the derivative of a function at a point. ● Be able to write an equation of a line using a derivative. ● Recognize alternate notations for derivatives and can use rules to compute basic derivatives. ● Connect limits to notions of continuity and differentiability of a function. ● Apply the product and quotient rules.
Unit 3: Differentiation: Composite, Implicite, and Inverse functions. Essential Questions:	Students will... Learning Targets: ● Be able to apply the chain rule.

• How are derivatives used to analyze the behavior of functions? • How is implicit differentiation used in finding derivatives?	• Compute derivatives of the remaining trigonometric functions. • Compute and use derivatives of implicitly defined function. • Calculate higher order derivatives and derivatives of inverse functions.
Unit 4: Contextual Applications of Derivatives Essential Questions: • How does velocity and acceleration relate to derivatives? • What does it mean when your limit is indeterminate? • Why does the derivative of an indeterminate limit work to help find the limit?	Students will... Learning Targets: • Interpret meaning of derivative in context • Connect position, velocity (speed), and acceleration to the appropriate derivative equation and apply in context. • Solve related rates problems. • Understand the concept of local linearity and can use it to construct a tangent line approximation. • Use derivatives and L'Hopital's Rule to evaluate Indeterminate limits.
Unit 5: Analytic Applications of Differentiation Essential Questions: • How can the derivative give us enough information to explain the behavior of the original function?	Students will... Learning Targets: • Draw conclusions about a functions behavior using the mean value theorem and extreme value theorem. • Use the first and second derivative tests to determine properties of functions. • Sketch graphs of functions and their derivatives. • Solve optimization problems.
Unit 6: Integration & Accumulation of Change Essential Questions: • How are geometry area formulas used to estimate area using limits? • How do integrals relate to real world applications involving area. • *What type of functions require a more advanced method to integrate?* • *Is there a function I will never be able to integrate without technology?*	Students will... Learning Targets: • Use numerical approximation to find the area under a curve (Trapezoidal Rule or Riemann Sums).. • Use Fundamental Theorem of Calculus to evaluate definite integrals. • Interpret behavior of accumulating function involving area under a curve. • Find antiderivatives and indefinite integrals using basic rules and notation. • Integrate using advanced methods need when finding the antiderivative from a chain rule (substitution, long division, and completing the square) • *Recognize and use Advanced integration techniques (integration by parts; linear partial fractions; improper integrals).*
Unit 7: Differential Equations Essential Questions: • How do differential equations help explain rates of change? • How are slope fields used to explain rates of change.	Students will... Learning Targets: • Verify solutions of differential equations. • Reason and work with slope fields. • Find general and particular solutions using separation of variables. • Understand the relationship between exponential models and differential equations. • *Understand the relationship between logistic models and differential equations.*
Unit 8: Applications of Integration	Students will...

Essential Questions: ● How are volumes evaluated using integrals. ● How can definite integrals be used to solve accumulation functions.	Learning Targets: ● Find the area between 2 curves expressed as functions of x or y. ● Find the volume with cross sections involving geometric shaped slices. ● Find the volume using Disk or Washer method given a function that revolves around any axis. ● Find the average value of a function on an interval. ● Use integrals ○ To connect position, velocity and acceleration. ○ In an applied context using the accumulating function and definite integrals. ● Use integrals to find arc length and distance traveled on a smooth curve.
Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions. Essential Questions: ● How is motion analyzed using parametric and vector equations? ● How is slope and area measured in a polar coordinate system?	Students will... Learning Targets: ● Apply the following with parametric equations: ○ Define and differentiate ○ Find the 2nd derivative ○ Find the length of curves ● Apply the following with vector-valued functions: ○ Define and differentiate. ○ integrate ● Apply the following to Polar Form ○ Define and differentiate ○ integrate ● Solve motion problems using parametric and vector-valued functions. ● Understand Polar Form and can find the area of a region bounded by 1 or 2 polar curves.
Unit 10: Infinite Sequence and Series Essential Questions: ● What do values from sequences and series tell me in application situations? ● In what situations would convergence be an important issue?	Students will... Learning Targets: ● Define convergent and divergent infinite series. ● Apply all the tests to determine convergence or divergence: ○ Geometric ○ Nth term ○ Integral test ○ P-series (Harmonic series) ○ Comparison tests ○ Alternating Series Test ○ Ratio Test ● Determine absolute or conditional convergence and find the alternating series error bound. ● Find the Taylor Polynomial Approximations of functions and apply the Lagrange Error Bound. ● Find the radius and interval of convergence of a power series. ● Find the Taylor or Maclaurin Series for a function. ● Represent functions as a power series.